International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN (P): 2319-3972; ISSN (E): 2319-3980
Vol. 10, Issue 1, Jan–Jun 2021; 29–36

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

STATIONARITY & HETEROSCEDASTICITY MODELS FOR ANALYZING STOCK MARKETS VOLATILITY

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ABSTRACT

Volatility in stock markets is a routine phenomenon. It has been an interesting and challenging area as well for the Stock Market investors since long. A lot of researches were carried out in the past taking help of various mathematical models but still it is an ongoing process. This study was undertaken to look into the various mathematical models applied to analysis the volatility in stock markets. Various equations, tests for stationarity, statistical tools and GARCH family models for heteroscedasticity were undertaken in the study.

KEYWORDS: *Stock Markets, Volatility, Stationarity, Heteroscedasticity, GARCH Models*

Article History

Received: ***06 Feb 2021*** **|** ***Revised:*** ***10 Feb 2021*** **|** ***Accepted:*** ***04 Mar 2021***

INTRODUCTION

Some people believe that stock market is a risky market to invest their hard earned money. On the other hand, others want to see the stock market bear and bull and to be part of that hassle. Now, this is all volatility in the market which makes this place unpredictable and risky one. It is believed that volatility and return are intermingled and volatility has an impact on returns of the stock market. *Mittal (2009)* analyzed volatility & return of the capital market of India from the perspective of understanding market behavior. It was observed that annual volatility declined from 2000 to 2007. Since then, however there was sharp increase in volatility. It is seen how the market responds to different happenings either positive or negative and how it effects the returns of the market. The BSE & NSE are two major stock exchanges in India as most of the share transactions are done by the investors in these two exchanges. Derivatives were introduced in the Indian stock market in 2001to control the volatility of Indian stock market. Bhatia & Jindal (2020) described various types & dimensions of volatility in stock markets. Nisha (2014) applied various mathematical & statistical models for analyzing stock market return and volatility in her study.

Statistical Tools used to Analysis the Trends in Stock Market Returns and Volatility Patterns

To calculate the returns, logarithmic difference between two periods is primarily taken by applying the following:

$$R_t(In\,P_t - In\,P_{t-1}) * 100 \quad \textbf{1}$$

Where Rt is the return in period t, Pt and Pt-1 are the daily closing prices of the index at time t and t-1 respectively.

Unit Root Test

For testing stationarity, let us consider an AR (1) model:

$$\gamma_t = p_1\gamma_{t-1} + \varepsilon_t \quad II$$

The simple AR (1) model indicated in above equation is classified a *random walk model*. In this AR (1) model if $|p_1|<1$, then the series is I(0) i.e. stationary in level, but if $p_1= 1$ then there exist what is called unit root problem. In other words, series is non-stationary. Most economists think that differencing is warranted if estimated p > 0.9; some would difference when estimated p > 0.8. Besides this, there are few formal means of testing for stationarity of a series.

Augmented Dickey Fuller Test

Dickey-Totaler test involve estimating regression equation and carrying out the hypothesis test. The simplest approach to testing for a unit root is with an AR(1) model. AR(1) process:

$$\gamma_t = c + p\gamma_{t-1} + \varepsilon_t \quad 3$$

Where *c* & ρ are parameters; is assumed to be white noise. If -1<p<1, then y is a stationary series while if ρ=1 , y is a non-stationary series. If the absolute value of ρ is greater than one, the series is explosive. Therefore, the hypothesis of a stationary series involves whether the absolute value of ρ is strictly less than one. The test is done by estimating an equation with y_{t-1}subtracted from both sides of the equation:

$$\Delta y_t = c + \gamma y_{t-1} + \varepsilon_t \quad 4$$

The DF test is valid only if the series is an AR(1) process; if the series is correlated at higher order lags, assumption of white noise disturbances is violated. The ADF controls for higher order correlation by the adding lagged difference terms of dependent variable to right-hand side of regression:

$$\Delta y_t = c + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \delta_2 \Delta y_{t-2} + \ldots. + \delta_p \Delta y_{t-p} + \varepsilon_t \quad 5$$

This augmented specification is then tested for in this regression.

$$H_0: \gamma = 0$$

$$H_1: \gamma < 0$$

The empirical research work done earlier has used this tool in their work (Kaur, 2004; Abdalla, 2012; Bordoloi& Shankar, 2008 and Karmakar, 2007).

Phillips–Perron Test

In statistics, the Phillips–Perron test (named after Peter C. B. Phillips and Pierre Perron) is a unit root test. That is, it is used in time series analysis to test the null hypothesis that a time series is integrated of order 1. It builds on the DF test of the null hypothesis $\delta = 0$ in

$$\Delta yt = \delta y_{t-1} + \mu t \quad 6$$

Where Δ is the first difference operator. Like the augmented Dickey–Totaler test, the Phillips–Perron test addresses the issue which process generating data for y_t might have a higher order of auto correlation than is admitted in test equation – making y_{t-1} endogenous & thus invalidating the Dickey–Fuller t-test. Whilst the augmented Dickey–Fuller test addresses this issue by introducing lags of Δy_t as regressors in test equation, the Phillips–Perron test makes a non-parametric correction to the t-test statistic. The test is robust with respect to the unspecified autocorrelation & heteroscedasticity in the disturbance process of test equation. (Kaur, 2004; Bordoloi& Shankar, 2008 and Karmakar, 2007) used the same for their research work.

Kwiatkowski–Phillips–Schmidt–Shin (KPSS) Test

In econometrics, Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests are used for testing a null hypothesis that an observable time series is stationary around a deterministic trend. The series is expressed as the sum of the deterministic trend, random walk & stationary error, the test is the Lagrange multiplier test of the hypothesis that the random walk has zero variance. KPSS type tests are intended to complement unit root tests, such as the Dickey–Fuller tests. By testing both the unit root hypothesis and stationarity hypothesis, one can distinguish series that appear to be stationary, series that appear to have a unit root & series for which the data (or the tests) are not sufficiently informative to be sure whether they are stationary or integrated. (Gupta and Basu, 2007), earlier literature mentioned the use of KPSS.

Autocorrelations and ACF (k)

Autocorrelation is one of statistical tools used for measuring the dependence of the successive terms in a given time series. Hence, it has been widely used to measure dependence in successive share price changes. Autocorrelation has been the basic tool used to test the weak form of EMH. The autocorrelation function ACF(k) for time series Yt and the k lagged series Yt-k is defined as:

$$ACF(K) = \left[\frac{\sum_{t=1-k}^{n}(Yt - Y)(Y_{t-k} - Y)}{\sum_{t=1}^{n}(Y_t - Y)^2}\right] \quad (7)$$

Where *Y* is considered as overall mean of series with n observations. The SE of ACF (k) is given by:

$$Se_{ACF(K)} = \frac{1}{\sqrt{n-k}} \quad (8)$$

When n is sufficiently large (n >50), the approximate value of standard error of ACF (k) is given by:

$$Se_{ACF(K)} = \frac{1}{\sqrt{n}} \quad (9)$$

To test whether ACF (k) is significantly different from zero, the distribution of t is used;

$$T = \frac{ACF(k)}{Se_{ACF(K)}} \quad (10)$$

As it is true for random walks, trends are also characterized by the extremely high autocorrelation. For both random walk series & series with trends, autocorrelation ACF (k) are very high & decline slowly as the lag value (k) increases. At the same time the ACF (k) of the first difference series (price changes or returns) are statistically insignificant when the series is a random walk series. A random walk series drifts up and down over time. In some situations it may be difficult to judge whether a trend or drift is occurring. Hence to determine whether a series has a significant trend or whether it is a random walk, the t-test is applied on series of first differences. (Bordoloi & Shankar, 2008).

Heteroscedasticity

One of most important issues before applying Generalized Autoregressive Conditional Heteroscedasticity (GARCH) methodology is to first examine residuals for evidence of heteroscedasticity. To test for the presence of heteroscedasticity in residuals of KSE index return series, the Lagrange Multiplier (LM) test for ARCH effects proposed by Engle (1982) is applied. In summary, test procedure is performed by first obtaining the residuals *t e* from ordinary least squares regression of conditional mean equation which might be an autoregressive (AR) process, moving average (MA) process or a combination of AR & MA processes; (ARMA) process. For example, in ARMA (1,1) process the conditional mean equation will be as:

$$r_t = \emptyset r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \qquad 11$$

After obtaining the residuals *t e* , the next step is regressing the squared residuals on a constant and q lags as:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \ldots + \alpha_q e_{t-q}^2 + v_t \qquad 12$$

The null hypothesis that there is no ARCH effect up to order q can be formulated as:

$$H_0 : \alpha_1 = \alpha_2 \cdots = \alpha_q = 0$$

Against the alternative:

$$H_1 : \alpha_i > 0$$

For at least one i = 1, 2, …, q

The test statistic for joint significance of q-lagged squared residuals is the number of observations times the R-squared (TR^2) from regression. $TR^2$is evaluated against χ^2□(q) distribution. This is asymptotically locally most powerful test.

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model

The conditional variance is represented as a linear function of its own lags in this model. The simplest model specification is the GARCH (1,1) model:

$$\textit{Mean Equation} \quad r_t = \mu + \varepsilon_t \qquad 13$$

$$\textit{Variance Equation} \quad \sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad 14$$

where $\omega > 0$ *and* $\alpha_1 > 0$ *and* $\beta_1 > 0$, *and*

r_t = *return of the asset at time t*

μ = *average return*

ε_t = *residual returns, defined as:*

$\varepsilon_t = \sigma_t z_t$

Where z_t is standardized residual returns (i.e. *iid* random variable with zero mean & variance 1), and α^2_t is conditional variance. For GARCH (1,1), the constraints $\alpha_{\geq 0}$ and $\beta_{1 \geq 0}$ are needed to ensure α^2_t is strictly positive. In this model, the mean equation is written as a function of constant with an error term. Since α^2_t is the one –period ahead forecast variance based on past information, it is called conditional variance. The conditional variance equation specified as a function of 3 terms:

- *A constant term :* ω
- *News about volatility from the previous period, measured as the lag of the squared residual from mean equation:* ε^2_{t-1} *(ARCH term)*
- *Last period forecast variance:* σ^2_{t-1} *(GARCH term)*

The conditional variance equation models time varying nature of volatility of residuals generated from mean equation. This specification is often interpreted in a financial context, where an agent or trader predicts this period's variance by forming a weighted average of a long term average (constant), the forecast variance from last period (GARCH term), and information about volatility observed in previous period (the ARCH term). If the asset return was unexpectedly large in either the upward or the downward direction, then the trader will increase the estimate of the variance for the next period. The general specification of GARCH is, GARCH (p, q) is as:

$$\sigma^2_t = \omega + \sum_{j=1}^{q} \alpha_j \varepsilon^2_{t-1} + \sum_{j=1}^{p} \beta_i \sigma^2_{t-1} \quad 15$$

Where, p is number of lagged α^2 terms and q is the number of lagged ε^2 term

The Exponential GARCH (E-GARCH) Model

This model captures asymmetric responses of time-varying variance to shocks & at same time, ensures that variance is always positive. It was developed with the following specification:

$$Ln(\sigma^2_t) = \omega + \beta_1 Ln(\sigma^2_{t-1}) + \alpha_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad 16$$

Where γ is asymmetric response parameter or leverage parameter. The sign of γ is expected to be positive in most empirical cases so that a negative shock increases future volatility or uncertainty while a positive shock eases the effect on future uncertainty. In macroeconomic analysis, financial markets & corporate finance, a negative shock usually implies bad news, leading to a more uncertain future. Consequently, shareholders would require a higher expected return to compensate for bearing increased risk in their investment. Above Equation is an E-GARCH (1,1) model. Higher order E-GARCH models can be specified in a similar way; E-GARCH (p, q) is as follows:

$$Ln(\sigma^2_t) = \omega + \sum_{j=1}^{p} \beta_j Ln(\sigma^2_{t-j}) + \sum_{i=1}^{q} \alpha_i \left\{ \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma_t \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \quad 17$$

The Threshold GARCH (T-GARCH) Model

Another volatility model commonly used to handle leverage effects is the threshold GARCH (or T-GARCH) model. In T-GARCH (1,1) version of model, the specification of conditional variance is:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad 18$$

Where d_{t-1} *is a dummy variable, that is :*

$$d_{t-1} = \begin{cases} 1\ if\ \varepsilon_{t-1} \leq 0 & bad\ news \\ 0\ if\ \varepsilon_{t-1} \geq 0. & good\ news \end{cases}$$

The coefficient γ is known as the asymmetry or leverage term. When γ = 0, the model collapses to the standard GARCH forms. Otherwise, when the shock is positive (i.e., good news) the effect on volatility is α_1, but when news is negative (i.e., bad news) effect on volatility is $\alpha_{1+}\gamma$. Hence, if γ is significant and positive, negative shocks have a larger effect on σ^2_t than positive shocks. This way, T-GARCH (p,q), the conditional variance equation is specified as follows:

$$\sigma_t^2 = \omega + \sum_{i-1}^{q}(\alpha_1 + \gamma_i d_{t\ 1})\varepsilon_{t-1}^2 + \sum_{j-1}^{p} \beta_j \sigma_{t-j}^2 \qquad 19$$

α_i, γ_i and β_j are non-negative parameters satisfying conditions similar to those of GARCH models.

(Akgiray, 1989; Ballie and DeGennaro,1990; Lamoureux and Lastrapes, 1990; Corhay and Tourani, 1994; Geyer, 1994; and Sakata and White, 1998) used ARCH, GARCH and their extension models in their research work.

CONCLUSIONS

The various mathematical, statistical & econometrics models considered in the study & depict the wide picture of analyzing the volatility of stock market. It included simple statistical methods those are used to forecast the volatility. Another important segment is of testing stationarity of the time-series data for which Unit Root Test, ADF Test and PP Test are applied. Econometric model i.e. KPSS Test had been very popular for testing a null hypothesis in the process of checking stationarity of data. Autocorrelation had been another important tool for measuring the dependence of successive terms in a given time-series proceeding volatility. Various GARCH family models are found very popular and significant to analysis the volatility of stock prices. All these models are extensively applied by the researchers in stock markets and still the search of further models is going on.

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